CALIBRATING VADOSE ZONE MODELS WITH TIME-LAPSE GRAVITY DATA: INTRODUCING A NEW TYPE OF CALIBRATION DATA

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Abstract
A change in soil water content is a change in mass stored in the subsurface, and when large enough, can be measured with a gravity meter. Over the last few decades there has been increased use of ground-based time-lapse gravity measurements to infer hydrogeological parameters. These studies have focused on the saturated zone, with specific yield as the most prominent target parameter. With a few exceptions, changes in storage in the vadose zone have been considered as noise. Here modelling results are presented showing that gravity changes will be measurable when soil moisture changes occur in the unsaturated zone. These results are confirmed by field measurements of gravity and georadar data at a forced infiltration experiment conducted over 14 days on a grassland area of 10 m by 10 m. An unsaturated zone infiltration model build in MIKE SHE is calibrated for saturated moisture content and the van Genuchten model parameter n using the gravity data, showing good agreement with the field data. The potential for gravity data to be used for the calibration of unsaturated zone model parameters is discussed.

Introduction
The water in the unsaturated zone constitutes only ~0.05% of the global amount of water, but is of great importance for plant’s uptake of water, biochemical processes, the global energy balance, and the precipitation balance and is in this way an important part of the hydrological cycle (D. A. Robinson et al., 2008). To reach the groundwater table, the precipitation must pass through the unsaturated zone. Soil saturation governs surface runoff during heavy rainfall in some areas. Thus, our ability to model water movements in the unsaturated zone is of great importance. Traditionally, borehole samples have been extracted and e.g. van Genuchten model
parameters determined through laboratory analysis (M. T. Van Genuchten, 1980). The disadvantage of this is that the samples are often disturbed by the extraction process (e.g. drilling) and are strictly speaking a point measurement, which is then assumed representative for the surrounding soil volume. Certain geophysical methods have the ability to address this problem: they are non-invasive and measure over a larger volume. Time-lapse gravity measurements are one of them.

A change in the soil water saturation constitutes a change in soil density. For sufficiently large changes over time, the change can be measured with modern gravity meters (gravimeters). This is a direct measure of the change in the water storage in the aquifer. The methods was first used to estimate the specific yield for an unconfined sandy aquifer back in 1971 (E. L. Montgomery, 1971). The interest for hydrogravimetric measurements has been small since, which can be ascribed to a large measurement uncertainty and that the method was considered too tedious to use. In the later years, instruments that are both more precise, robust and easier to use, have arrived on the market. It has led to a renewed interest in gravimetric measurements for hydrological purposes. Focus has been on the saturated part of the soil, where variations in gravity of over 100 microGal (1 µGal = 10^{-8} \text{ ms}^{-2}) have been observed (D. S. Chapman et al., 2008; J. F. Howle et al., 2003; D. R. Pool, 2008). Meanwhile, software to calculate the gravity signal from both saturated (S. Leiriaro et al., 2009) and unsaturated (M. A. Ribers, 2009) zone models have been developed. This enables the inclusion of time-lapse gravity data in a model calibration in line with data for e.g. water table variations.

The HYDROGRAV research project (www.science.hydrograv.dk) is a collaboration between DTU Environment, DTU Space, DHI and COWI. The aim of the project is to investigate the use of gravity data in hydrological monitoring, with the specific objective to facilitate the use of gravity data in model calibration. This is done through ground-based measurements (the topic of this paper) and with data from the GRACE satellites. The satellites provide data with a resolution of several hundred kilometers every tenth day and are used for modeling on a regional scale. We have earlier with field data shown that ground-based gravity measurements provide a significantly better model parameter estimation on a model for water storage in a river bank (L. Christiansen et al., 2008). The improvements are in particular seen for specific yield.

The contribution to gravity from the unsaturated zone has often been disregarded in previous investigations. Model results show that seasonal variations alone can be more than ten microGal (M. A. Ribers, 2009), which is a measureable signal. The idea to infer model parameters for the unsaturated zone through a field experiment was thus natural. In the fall 2009, we conducted an experiment with forced infiltration over roughly 100 m^2 at Arrenæs on the Danish island Sjælland. Changes in gravity over time were measured. Since the inversion of gravity data to a density distribution is non-unique, we chose to combine the gravity measurements with data from cross-borehole georadar. A similar experiment with ground penetrating radar (GPR) in combination with geoelectrical measurements
has previously been conducted at Arrenæs (T. M. Hansen et al., 2008; M. C. Looms et al., 2008a; M. C. Looms et al., 2008b).

We here present the measurement methods and the used setup in the field. We show that there is a good correspondence between gravity- and radar data. A MIKE SHE model for the unsaturated zone is calibrated for saturated moisture content and the van Genuchten model parameter n.

The work presented is part of a PhD-project at DTU Environment and should be seen as “work in progress”. The dissertation will be available at www.env.dtu.dk/Publikationer.aspx as of mid October 2010 and by direct contact to Lars Christiansen (larc@env.dtu.dk). The main results are expected to be published as a series of papers in scientific journals in 2011.

**The theoretical gravity signal from water**

An often used method to calculate the gravity signal from a change in the soil’s water saturation is to view the change as the addition (removal) of an horizontally infinite slab of water with the thickness $\Delta z$, a Bouguer slab. No matter the distance from the gravimeter to the water, the change in gravity can be calculated as

$$g_{\text{vand}} = 2\pi G \rho \Delta z \Delta \theta$$

where $G = 6.673 \times 10^{-11}$ m$^3$kg$^{-1}$s$^{-2}$ is the gravitational constant, $\rho$ is the density of water and $\Delta \theta$ is the change in soil water content. For one meter of water ($\Delta z = 1$, $\Delta \theta = 1$) one get $g_{\text{water}} = 42$ µGal. This method is often very convenient for a preliminary estimation of whether a given amount of water will give a measureable gravity signal. For more complex geometries, a division of the soil into prisms, for which the exact gravity can be calculated individually, can be used as described by S. Leiriaro et al. (2009) and M. A. Ribers (2009). For groundwater models that divide the domain into prisms, the gravitational response due to a change in water content can be calculated for any position to any time in the model.

**Measuring gravity changes**

Gravity measurements can be done with absolute and relative gravimeters, respectively. The latter is cheaper, more mobile, and easier to operate and is therefore more common. DTU Space has two relative gravimeters of the type Scintrex CG-5 Autograv. The instrument has a resolution of one µGal and a typical repeatability of five µGal. Compared to older models, e.g. the for some well-known LaCoste & Romberg gravimeter, the CG-5 is more robust and data are less affected by the user’s ability to operate the instrument (see Fig. 1).

Due to instrument technical reasons, relative gravimeters have an intrinsic drift, which varies with time. For relative gravimetric measurements a reference station where no (significant) change in gravity occurs during the time of the field campaign is needed. The drift, which is often assumed linear, is determined be visiting the reference station repeatedly during a
series of measurements. One “measurement” of the gravity difference between the reference point and another point in the network consists of a number measurements, for which the mean and standard deviation are subsequently found. When the change in gravity over time is of interest, the temporal change in the gravity difference between a point of interest and the reference station is investigated. In the setup used here, a full measurement of the gravity difference took one hour.

![Image of CG-5 relative gravimeter on tripod](image)

Fig. 1: The CG-5 relative gravimeter on its tripod, which is used to level the instrument. The tripod here further rests on a metal frame, which has been cast 0.4 m into the soil in order to create a stable foundation, under which the water is still able to flow into the soil. In other places in the field we used 0.4 x 0.4 x 0.4 m³ concrete blocks cast directly into the ground. Behind the instrument the irrigation tubes are visible.

Ground penetrating radar

Cross-borehole GPR measures the time it takes for an electromagnetic wave to travel through the soil from the transmitter antenna in one borehole to the receiver antenna in another borehole. Since the distance between the two boreholes is known, one has a measure of the wave velocity. The velocity depends on the general composition of the soil and the water saturation $\theta$ (G. C. Topp et al., 1980). For sand the relationship is well known and $\theta$ can be determined.

We performed the GPR-measurements according to the method used by M. C. Looms et al. (2008b). We measured both with and without vertical displacement of the antennas, which gave 2D profiles and 1D distributions, respectively, of the soil water content in 0.25 m intervals.

The field site

We chose to use a site at Arrenæs, a headland in the Arresø lake on the Danish island Sjælland (Sealand). The soil there consists of glacial deposits
with large quantities of fine-to-medium sand. Based on hand augering, hand-dug holes, surface-GPR, drilling records and direct-push sampling, we have a knowledge of the overall geology at the site is. An important characteristic is the sudden horizontal changes exemplified by a one meter thick clay layer close to the surface, which was observed to end abruptly within few meters. Even small variations in the geology can have a large effect on water infiltrating though the unsaturated zone. Based on the direct-push sampling, van Genuchten model parameters have previously been determined in the laboratory for different geological layers at the site (M. Bakmand-Mikalski and Karlsson, I.B., 2008).

The groundwater table is located 25-30 m below surface at the site, which gives a thick unsaturated zone over which to measure. The area was until the year-end 2009 used as a test area for København Energi (Copenhagen Energy), who used forced irrigation of water from the surrounding lake to clean the polluted water. The surface was grass-covered during our experiment.

**Experiment setup**

We set up a drip irrigation system to irrigate 10.33 x 10.33 m² with 86 mm of water per day. This corresponds to a total flux of 6.5 liters per minute. See Fig. 2. The exact choice of where to irrigate was a balance between the expected geology (we wanted to avoid clay layers), distance to the neighbors and the existing irrigation plant, and the surface inclination. The chosen spot thus has an average inclination of 6%. For the same reason, we chose to supplement the main gravity measurements, which took place at the central gravity station (station 1), with occasional measurements at the stations above and below the irrigated area (stations 2 and 4).
A first interpretation of data

The data series from station 1 and 4 are shown in Fig. 3. From station 2, only a few measurements exist which we have left out here. We observe that gravity changes significantly over the first 4-6 days, after which the curves reach a plateau. We had initially, based on an assumption of 1D vertical flow, expected a larger signal at station 1 than at station 4, since more water is infiltrated within the sensitive volume for station 1 than for station 4. That the signals are comparable in size, we ascribe to a large amount of water flowing horizontally through the soil down past station 4, which is placed downhill. GPR-data further indicate preferential flow starting around 5 m below the surface. If this is the case, water will drain rapidly through these “channels” and leave the sensitive volume of both the gravity and the GPR-measurements. Fig. 4 shows the average water content for the soil in-between the four boreholes in 0.25 m intervals. The 2D-profiles are not shown here.

Since gravity and GPR-data describe the movement of the same mass down through the soil, the two data types must show comparable results. In order to be able to compare, the GPR-data was recalculated into changes in gravity over time. As is seen from Fig. 5, there is a good correspondence between the two data types. It confirms that the signal in both data types is due to the change in soil water content. For the gravity signal, this is especially important, since there is a low signal-to-noise ratio in this data set and an unexplained correlation between the measurements at stations 1
and 4. The figure further show the modeled signal from a layered 1D MIKE SHE model based on the above mentioned van Genuchten parameters derived in the laboratory. The model does not allow for horizontal or preferential flow, which explains the discrepancy between the model and the data after day 6.

Data after the irrigation period show signs of strong noise. They are therefore not used in the parameter estimation.

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**Fig. 3:** The gravity signal at station 1 (diamond) and station 4 (filled square), respectively. Changes are with respect to the average gravity before irrigation started. A correlation between the two data series, for which we at the present do not have an explanation, is seen. Error bars show one standard deviation uncertainty.
Fig. 4: Mean soil water content for the soil between the four boreholes. Solid line is the initial water content. Dashed lines are, from left to right, water content on day 3, 4, 6 and 9, respectively. Only selected days are shown.

Fig. 5: Changes in gravity at station 1 (diamond), GPR-data recalculated into changes in gravity (stars) and the modeled gravity signal based on a 1D layered model (grey line). From day 10 and on we used a different GPR-instrument, which gave rise to a displacement in the measured $\theta$.

**Sensitivity analysis**

Since the aim is to estimate model parameters for a hydrological model, we have investigated how changes in model parameters affect the modeled
gravity signal. The larger a change in gravity due to a change in a specific parameter is, the better we can expect to be able to constrain the parameter through gravity observations. The sensitivity on gravity due to a change in parameter value changes with time and space in a model. In Fig. 6 we have plotted the changes gravity due to a 5% increase in each model parameter, one at a time, for station 1. We observe that the gravity is most sensitive to $\theta_s$, $n$ and $K_s$.

![Fig. 6: Absolute sensitivity due to a 5% change in parameter values as compared to the baseline values. $\theta_s$ is the saturated water content, $\theta_r$ is the residual water content, $K_s$ is the saturated hydraulic conductivity, and $n$ and $\alpha$ are van Genuchten fitting parameters.](image)

**Calibration results**

We proceed to present the calibration results. For full details, please refer to the above announced PhD dissertation.

MIKE SHE does not allow for horizontal flow. As a way to model the disappearance of water to outside the sensitive volume for the gravity measurements at station 1, we only modeled the change in gravity down to 5 m below surface. The rest of the water was simply disregarded. For the calibration, an interface was build to enable calibration of MIKE SHE with the general parameter estimation software PEST (J. Doherty, 2004). We used the above mentioned parameters estimated through laboratory test for parameters not calibrated.

Based on the sensitivity analysis, we calibrate for $\theta_s$, and $\theta_s$ and $n$, respectively. Fig. 7 and Fig. 8 show the estimated parameter values with their 95% uncertainty bounds and the corresponding calibration curve. The normalized root mean square error (NRMSE) is used as a measure of fit between mode and data. A value of one corresponds to an average model misfit equal to the average standard deviation on field data. Fig. 9 and Fig. 10 show the results when calibrating for $\theta_s$ and $n$. As is to be expected, the uncertainty on the value for $\theta_s$ is larger than when calibrating for only one parameter. The estimated value is slightly higher, but are in both cases in the range of what can be expected for a mainly sandy soil.
Fig. 7: Calibrated value for $K_s$ with 95% error bounds. Scale chosen to match Fig. 9 for better comparison.

Fig. 8: Calibration curve, when calibrating for $\theta_s$. Only data marked with blue were used in the calibration. The rest (black) were discarded due to noisy data.

Fig. 9: Calibrated values for $\theta_s$ and $K_s$ with 95% error bounds.
Discussion and conclusion

Time-lapse gravity is a non-invasive and non-destructive measurement method, which provides data for a larger volume than what is obtained through tests on borehole samples. We have shown that the infiltration of water gives a measureable signal and shown that gravity data can constrain model parameters for the unsaturated zone. The present data set suffers from a low signal-to-noise ratio, which limits a more precise parameter estimation.

To collect gravity data can be relatively time consuming, but it is balanced by the fact that no other method is able to do the same: to provide a direct measure of the change in soil water content. The challenge is that the gravity data do not in themselves show where the water is. The measurements should thus in most cases be combined with hydrological modeling, which constrains the movement of the water and thereby the possible solutions to the inverse gravity problem. This methodology is termed coupled hydrogeophysical inversion (A. C. Hinnell et al., 2010).

The limitation in identifying where the water is partly lies in that we measure from the surface. A way to obtain a better spatial resolution is to measure gravity down a borehole. A further advantage of this would be an improved ability to discern the gravity signal from the saturated and the unsaturated zones, respectively. A new gravimeter, which might be suitable for this task, is under development at Scintrex Ltd. in Canada.

Another potential use of gravity measurements is the systematic long-term monitoring of water resources on a basin scale. The challenge here is to find a gravimetrically stable reference point. There are several option for this, e.g. to tie the network to a point monitored by an absolute gravimeter.

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