A MODEL FOR LONG-TERM DISTRIBUTION OF WAVE INDUCED LOADS IN STEEP AND BREAKING SHALLOW WATER WAVES

Hans Fabricius Hansen  
DHI  
DK-2970 Hørsholm, Denmark  
hfh@dhigroup.com

Iris Pernille Lohmann  
DNV  
DK-2900 Hellerup, Denmark

Jacob Tornfeldt Sørensen  
DHI  
DK-2970 Hørsholm, Denmark  
jts@dhigroup.com

Flemming Schlütter  
DHI  
DK-2970 Hørsholm, Denmark  
fls@dhigroup.com

ABSTRACT
A new approach to determine the design wave load on bottom-fixed structures in shallow water breaking waves is presented here. The method takes into account the effects that wave breaking has on both the wave height distribution and the wave induced loads on the structure. The loads on offshore wind turbine foundations in irregular seas with a significant amount of wave breaking are modeled in a physical wave tank. The loads are related to wave characteristics as steepness and Ursell number, and a non-linear transfer function between wave height/period and wave load is established. Characteristic historical load events are now established by combining the transfer function with a record of the wave climate at the site. The latter is taken from a hindcast database, but could also come from site measurements. The long-term distribution of the load is estimated by adopting traditional extreme value analysis techniques to the historical characteristic loads.

INTRODUCTION
Hundreds of offshore wind turbines have been installed in shallow water in recent years and many more are coming up in the near future. Low water depths significantly reduce the cost of the foundations, compared to intermediate water depths but it also introduces an additional design challenge as wave breaking induced loads now become a factor to consider.

The maximum inline force and overturning moment on a bottom-fixed shallow water structure in a design sea state is likely to be composed of drag and inertia forces and a load induced by wave slamming on the structure. While the former two are well described by use of an appropriate regular wave theory and the Morison's equation, the latter may be difficult to determine, as it will depend on the type of wave breaking and the size of the breaking wave. Moreover, the slamming load may be significantly larger than the drag and inertia load. All this obviously leads to a large uncertainty in the maximum load for which to base the design of a structure at a site where some degree of wave breaking is to be expected over the design life of the structure.

In this paper an adaption of the method proposed by Tromans and Vanderschuren (1995) has been applied to determine the long-term distribution of the maximum inline force. First the relationship between wave height and period and the inline force is examined from physical model tests. Secondly, the characteristic maximum inline force expected on a foundation in a rather exposed shallow water site is computed for historical storms using spectral hindcast data. Finally, extreme value analysis is performed on these characteristic loads and a central estimate of the long-term distribution of the inline force obtained.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>Proportionality coefficient of wave breaker index</td>
</tr>
<tr>
<td>( F_x )</td>
<td>Inline force</td>
</tr>
<tr>
<td>( F_{\text{mp}} )</td>
<td>Most probable maximum inline force in the main wave direction</td>
</tr>
<tr>
<td>( F_{\text{max}} )</td>
<td>Maximum inline force in the main wave direction</td>
</tr>
<tr>
<td>( h )</td>
<td>Water depth</td>
</tr>
<tr>
<td>( H )</td>
<td>Wave height</td>
</tr>
<tr>
<td>( H_b )</td>
<td>Wave height at breaking</td>
</tr>
</tbody>
</table>
**EXPERIMENTAL SETUP AND WAVE CONDITIONS**

Physical model tests have been carried out to determine the loads on a monopile foundation in shallow water on a sloping seabed. Wave gauges have been deployed throughout the test basin and forces on the monopile measured with a 3D force gauge at the monopile/seabed interface. The experiments are more thoroughly described in Nielsen et. al. 2012.

All experimental results referred in the present paper have been scaled to a typical prototype scale. The scaling factor is 1:36.6.

![Figure 1. Sketch of experimental setup. Not to scale.](image)

Results from model tests with a prototype monopile foundation of 6 m diameter placed in 20 meters of water depth on a seabed slope of 1:25 are presented in this paper. Only tests with unidirectional waves have been considered at present. The frequency spectra of the considered model tests are shown in Figure 2 and the spectral characteristics given in Table 1.

![Figure 2. Model test spectra.](image)

**WAVE HEIGHT DISTRIBUTION**

The short-term wave height distributions in the physical model tests have been examined by means of zero-downcrossing analysis. The observed distributions have been plotted in Figure 3 along with the theoretical and empirical distribution functions from literature. Forristall (1978) have proposed the following distribution based on measured hurricane waves in the Gulf of Mexico

\[
F(H) = 1 - \exp\left[-\left(\frac{H}{0.681H_{m0}}\right)^{2.126}\right]\tag{1}
\]

The distribution proposed by Battjes and Groenendijk (2000) is based on laboratory experiments with waves on shallow foreshores and takes into account the effect of wave breaking on the wave height distribution. It is a composite Weibull distribution given by:

\[
F(H) = \begin{cases} 
F_1(H) = 1 - \exp\left[-\left(\frac{H}{H_1}\right)^2\right], & H \leq H_{tr} \\
F_2(H) = 1 - \exp\left[-\left(\frac{H}{H_2}\right)^{3.6}\right], & H > H_{tr}
\end{cases}
\tag{2}
\]

where the transitional wave height is given by \(H_{tr} = (0.35 + 5.8\tan(\theta))h\). Reference is made to Battjes and Groenendijk (2000) for details on how to obtain \(H_1\) and \(H_2\).

A thorough account of available deep- and shallow water wave height distributions is given in Rattanapitikon (2010).

From the distribution plots in Figure 3 it is observed that the Rayleigh distribution overestimates the wave heights, perhaps with the exception of test #2, while the Forristall wave height distribution is found to give a good representation of the measured wave heights. This is an indication that the surface elevation process is too broad-banded for the narrow-band assumption of the Rayleigh distribution to hold.

However, there is a tendency that the Forristall distribution underestimates the intermediate wave heights and overestimates the very largest wave heights for test #3 and #4, ie. the tests with the most severe sea states. This indicates that a shallow water wave height distribution, such as the Battjes and Groenendijk distribution, may be more appropriate although it does not provide a significantly better fit in this case.

---

**Table 1. Model test characteristics**

<table>
<thead>
<tr>
<th></th>
<th>(H_{m0}) [m]</th>
<th>(T_{01}) [s]</th>
<th>(T_{02}) [s]</th>
<th>(T_p) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test #1</td>
<td>4.6</td>
<td>8.3</td>
<td>7.5</td>
<td>10.1</td>
</tr>
<tr>
<td>Test #2</td>
<td>6.3</td>
<td>8.8</td>
<td>7.8</td>
<td>10.8</td>
</tr>
<tr>
<td>Test #3</td>
<td>8.1</td>
<td>8.7</td>
<td>7.7</td>
<td>10.1</td>
</tr>
<tr>
<td>Test #4</td>
<td>8.5</td>
<td>10.1</td>
<td>8.7</td>
<td>12.6</td>
</tr>
</tbody>
</table>
Figure 3. Wave height and inline load distributions from physical model tests. $H_{m0} = 8.2\,m$, $T_p = 12.6\,s$.

**INLINE LOADS**

The inline wave loads are dominated by inertia forces rather than drag forces for the tested wave conditions. This means that the maximum load in the direction of wave propagation will occur before the passing of the wave crest. This is also seen from Figure 4.

Figure 4. Time series of surface elevation 100 meters sideways of the foundation and simultaneous inline load

A zero down-crossing analysis of the measured time series of surface elevation has been carried out and wave heights $H$ and periods $T$ of the individual waves determined. The maximum load in the direction of wave propagation measured during each wave cycle from down-crossing to down-crossing has also been extracted. The maximum load is plotted against the zero-crossing wave height in Figure 5 for the four physical model tests with significant wave heights ranging from 4.6 m to 8.5 m.

Figure 5. Maximum inline load as a function of wave height. Color codes refer to different tests with varying wave conditions.
It is observed in Figure 5 that the relationship between wave height, $H$, and inline load, $F_x$, is linear for low wave heights. This is expected, as the loads are inertia dominated and the load therefore proportional to the acceleration. However, for larger wave heights, the maximum inline force increases dramatically although the dependency on $H$ becomes less clear. This is believed to be caused by wave slamming from breaking waves, an assumption that is further supported by the time series example shown in Figure 4. The large inline load peak at around 925 seconds is associated with a very steep wave front, and the peak load is observed right at the moment of the wave front passage.

Wave breaking is influenced by wave steepness and the wave height to water depth ratio and, as seen from Figure 6, the inline force can also be related to these parameters. Figure 6 shows the inline force as a function of relative depth ($h/L_0$) and wave steepness ($H/L_0$), where the deep-water wave length, $L_0 = g/(2\pi)^2$, is calculated from the period of the individual waves determined by zero-crossing analysis.

The breaking height, $H_b$, is expressed by Goda (2010) as

$$ \frac{H_b}{L_0} = A \left( 1 - \exp \left( -1.5 \frac{\pi h}{L_0} \left( 1 + 11 \tan^{4/3} \theta \right) \right) \right) $$

(3)

where $\theta$ is the seabed slope and $h$ is the water depth. Wave breaking in irregular seas will occur for values of the proportionality index $A$ in the range of 0.12 to 0.18. The breaking wave height has been plotted for $A=0.12$ and $A=0.18$ in Figure 6.

The breaking height, $H_b$, is expressed by Goda (2010) as

$$ \frac{H_b}{L_0} = A \left( 1 - \exp \left( -1.5 \frac{\pi h}{L_0} \left( 1 + 11 \tan^{4/3} \theta \right) \right) \right) $$

(3)

where $\theta$ is the seabed slope and $h$ is the water depth. Wave breaking in irregular seas will occur for values of the proportionality index $A$ in the range of 0.12 to 0.18. The breaking wave height has been plotted for $A=0.12$ and $A=0.18$ in Figure 6.

It is seen that the inline force is increasing as the waves approach the breaking height. However, the inline force is also dependent on the absolute wave height as low but steep waves are unlikely to cause the same load as high, steep waves. This may be expressed as a dependency on the wave Ursell number;

$$ Ur = \frac{H \cdot L_0^2}{h^3} $$

(4)

**TRANSFER FUNCTION**

A transfer function that expresses the inline load as a function of the proportionality index, $A$, and the Ursell number, $Ur$, of the individual waves is established. $A$ is obtained by reordering eq. (3) and replacing $H_b$ with the height of the individual waves $H$, ie.

$$ A(H, L_0) = \frac{H}{L_0} \left( 1 - \exp \left( -1.5 \frac{\pi h}{L_0} \left( 1 + 11 \tan^{4/3} \theta \right) \right) \right) $$

(5)

The inline load as a function of $A$ and $Ur$ is modeled as the sum of an average load and a stochastic variance about the average. Both average and standard deviation are therefore calculated for binned values of the measured forces. The ‘binning’ can be carried out in many ways in the two-dimensional domain of $A(H, L_0)$ and $Ur(H, L_0)$. However, it was found that the results were not very sensitive to the way in which data were binned. The adopted methodology consists in calculating the mean and standard deviation of the inline load for each observation by taking the 50 nearest observations. The nearest points are found in $(\log_{10}(Ur), 20 \times A)$ space, in order to get similar scales of $Ur$ and $A$. The values for the regression of the transfer functions and the transfer functions themselves are shown in Figure 7. The data show that not only the average load but also the standard deviation is increasing for increasing values of $A$ and $Ur$.

![Figure 6](image)

Figure 6. Maximum inline force plotted against relative depth, $h/L_0$ (x-axis) and wave steepness, $H/L_0$ (y-axis), where $L_0$ is the deep-water wave length.

![Figure 7](image)

Figure 7. Measurement data (points) and fitted transfer function (contours) of average load (left) and standard deviation of load (right), as functions of $A$ and $Ur$. 

4 Copyright © 2012 by ASME
The following 6-parameter transfer function is found to fit well to data of both average and standard deviation.

\[
F_{x(avg,STD)}(A,Ur) = \left\{a_1 \times \exp(a_2(A - a_3))\right\} \times \{b_1 + b_2 \times Ur^{b_3}\}
\]  

(6)

The parameters of the transfer function are found from the data plotted in Figure 6 by least-square minimization. The estimated parameter values are given below.

**Table 2. Parameters of transfer function, eq. (6).**

<table>
<thead>
<tr>
<th></th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(a_3)</th>
<th>(b_1)</th>
<th>(b_2)</th>
<th>(b_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>27.21</td>
<td>13.91</td>
<td>-0.227</td>
<td>-0.633</td>
<td>0.990</td>
<td>0.0758</td>
</tr>
<tr>
<td>STD</td>
<td>18.49</td>
<td>17.09</td>
<td>-0.003</td>
<td>0.306</td>
<td>2.401</td>
<td>0.1795</td>
</tr>
</tbody>
</table>

**SIMULATION OF SHORT-TERM DISTRIBUTIONS**

The transfer function can be used to simulate the inline loads on the foundation in a specified sea state if the distribution of individual wave heights and wave periods within the sea state is known. The wave heights could be obtained by sampling from a theoretical or empirical wave height distribution such as the Rayleigh distribution or the Battjes and Groenendijk (2000) distribution. Theoretical models also exist for the joint density of wave heights and periods, for instance Longuet-Higgins (1983) which is based on the assumption of a narrow-banded process. However, this model does not give a good fit to standard wind sea spectra which are too broad-banded (Ditlevsen, 2002). Hence resort is taken to linear simulation of the surface elevation process from the wave spectrum. The individual waves are identified from the surface elevation process by a zero-crossing analysis.

The parameters \(A\) and \(Ur\) are calculated for all the waves and the transfer function (6) applied to calculate the inline loads. The stochastic part of the load is assumed to be normal distributed around the average load value and the stochastic load contribution is randomly sampled for each individual wave cycle using the standard deviation transfer function (Table 2).

Figure 8 shows the measured and simulated inline load distributions. The transfer function has also been applied directly to the measured waves (dash green). In this way all uncertainty related to the simulation of wave heights and wave periods is eliminated and the performance of the load transfer function can be assessed directly. It is noted that the transfer function is generally performing well on the measured waves.

Two sets of simulated loads based on simulated waves are also shown in Figure 8 – one based directly on the simulated waves (dash-dot red) and one set in which the wave heights of the steepest waves have been corrected so that the value of \(A(H,L_0)\) does not exceed 0.18 (dash blue). It should be noted that the wave spectrum energy is not conserved by this rather crude correction. The correction is found to yield slightly reduced loads at the 1 in 1000 level, although the difference is not significant for the tested sea states.

![Figure 8. Measured and simulated distributions of inline loads](image-url)
established departing only from the wave frequency spectrum. The wave frequency spectrum is often directly available from spectral wave hindcast models or can be estimated from parameters such as $H_m$ and $T_p$ applying a standard wind-sea spectrum such as for instance the JONSWAP spectrum.

**LONG-TERM LOAD DISTRIBUTION**

In the previous section we showed how we can estimate the short-term distribution of inline wave loads for a random sea state characterized only by its wave frequency spectrum. We will now use this information together with an adaption of the Tromans and Vanderschuren method to derive the long-term distribution of inline loads at a particular site.

The event-based method proposed by Tromans and Vanderschuren (1995) to determine the long-term distribution of the maximum wave height has gained wide acceptance within the offshore industry. Advantages of this method are; 1) the storms duration is inherently accounted for, 2) the effect of the short-term variability of the surface elevation process on the long-term extremes is included. The method consists in the following:

- calculate the most probable maximum wave height for each storm event in the metocean database
- perform a traditional extreme value analysis on the set of most probable maximum wave height.
- convolution of the long-term distribution of most probable wave heights with the short-term variability of the maximum wave height conditional on its most probable value

The method is here used directly on the inline load rather than on the individual wave height. Individual waves are obtained for storms in the metocean database by linear simulation of the surface elevation process. Inline forces are calculated using the transfer function (6) and the maximum for each storm is extracted. The procedure is repeated a number of times with different random number seeds in order to estimate the distribution of the maximum inline load for the storm. The distribution is well approximated by a generalized Gumbel distribution, where the $\alpha$ and $\beta$ parameters are estimated by least-square regression;

$$P(F_{\text{max}} < f) = \exp \left( -\exp \left( -\ln(N) \left( \frac{f}{\alpha} \right)^\beta - 1 \right) \right)$$  \hfill (7)

The value of the $N$ parameter results from the calculation of the most probable maximum wave height in the storm and is a measure of the number of waves in the storm. Figure 9 shows an example of the estimation of the most probable inline load in a storm. The empirical cumulative distribution of the maximum loads extracted from each random simulation is shown with dots in the top panel and the corresponding histogram in the lower panel. The generalized Gumbel distribution (7) with parameters estimated by linear regression is plotted on top of this.

The most probable maximum inline load in the storm is the mode, $\alpha$, of the Gumbel distribution, and the shape parameter, $\beta$, is a measure of the variance of the inline load for the particular storm and is allowed to take any value larger than 0. It is noted that Tromans and Vanderschuren (1995) used a fixed value of 2.
EXAMPLE APPLICATION

An example of how the inline load model can be applied to determine long-term loads at an exposed shallow water site is given here. 31 years of wave hindcast has been extracted from DHI’s North Sea/Baltic Sea hindcast database for a location close to the Horns Rev I and II offshore wind farms, located off the western coast of Jutland. The extraction point is located at a depth 22 m MSL and its location is indicated in Figure 12. The wave climate is rather severe, with significant wave heights reaching ~9 m during storms as shown in the insert in Figure 12.

Declustering of the time series of significant wave heights into storms has been made by requiring that individual storm peaks are at least 24 hours apart and that the significant wave heights between consecutive storms has been below 80% of the lower of consecutive storm peaks.

The storm modes (i.e. the most probable maxima) for both wave heights and inline loads are computed for the largest storms from the random simulations. The wave height has also been computed using the Battjes and Groenendijk wave height distribution.

A traditional Peak-Over-Threshold extreme value analysis is carried out on the most probable wave heights, $H_{mp}$, and the most probable inline loads, $F_{mp}$, to determine the long-term distribution of these. The short-term variability must also be accounted for by convolution of the long-term storm density distribution with the short-term distribution of maxima in order to arrive at the long-term distribution of the maximum wave height and inline load. The expression for the long-term distribution of the maximum wave height is given by Tromans and Vanderschuren (1995), while the expression for the long term distribution is given below:

$$p(F_{\text{max}} < f) = \int_{F_{\text{mp},\text{min}}}^{\infty} p(F_{\text{max}} < f | F_{\text{mp}}) p(F_{\text{mp}}) dF_{\text{mp}}$$

$$= \int_{F_{\text{mp},\text{min}}}^{\infty} \exp\left(-\exp\left(-\ln(N)\left(\left(\frac{f}{F_{\text{mp}}}\right)^{\beta} - 1\right)\right)\right) p(F_{\text{mp}}) dF_{\text{mp}}$$

(8)

It is noted that while (7) expresses the distribution of the maximum inline load in a single storm, (8) expresses the long-term distribution of the maximum inline load.

Long-term distributions of both maximum individual wave height and inline load based on linear simulation are shown in Figure 10 and Figure 11 respectively. The extreme wave estimated from linear simulation is probably too high for this shallow water site, with a wave height to water depth ratio in excess of 0.8 for the 100 year wave. This indicates that a shallow water wave height distribution such as for instance the Battjes and Groenendijk distribution may be more appropriate. Adopting this distribution would require that a correction was applied to the wave heights from the linear simulation so that they emulate a sample of wave heights in the Battjes and Groenendijk distribution.

It is also observed that the difference between the dashed and the solid line is much larger for the inline force in Figure 11 compared to the wave height in Figure 10. This is expected, as the variability of the inline force is much larger than of the wave height, something which is also clearly seen by comparing Figure 3 with Figure 8. The probability of occurrence of large waves decreases much more rapidly than the probability of occurrence of large inline forces, i.e. the inline force probability density has a larger variance.

---

Figure 10. Weibull distribution (solid line) fitted to the most probable wave heights of hindcast storms (+). The long-term distribution of individual wave height, $H$, obtained after convolution with the short-term distribution is shown with a dashed line. Return period in years.

Figure 11. Weibull distribution (solid line) fitted to the most probable inline load in hindcast storms (+). The long-term distribution of inline force, $F_x$, obtained after convolution with the short-term distribution is shown with a dashed line. Return period in years.
DISCUSSION AND CONCLUSIONS

It is shown that depth-induced wave breaking is likely to reduce the maximum wave heights in shallow water. Wave height distribution models that take into account shallow water effects, such as for instance the Battjes and Groenendijk distribution may therefore be more appropriate than the standard Rayleigh distribution or the Forristall distribution to describe the maximum wave heights to expect at a shallow water site. However, if shallow water effects are accounted for in the design wave heights at the site, one must also consider the adverse effect of wave breaking on the loads on partly submerged structures at the site. Physical model tests have shown that these loads may account for more than half the total inline load on the foundation. As they act on the foundation at the top of the wave crest, they are also significantly impacting the total overturning moment on the foundation.

It is shown that the variance of the inline load is considerably larger than that of the wave height itself. A statistical approach to determining the long-term distribution of the inline load accounting for the large spreading is proposed here. The method is an adaption of the widely recognized method by Tromans and Vanderschuren (1995) to determine the long-term distribution of individual wave or crest heights from records of significant wave height or spectral moments.

An example application of the method is given, and it is observed that it is important to account for the short-term variability of the inline load.

The method could easily be used for estimating other loads, such as for instance the overturning moment on the foundation. The transfer function giving the relationship between wave height/period and load can also be established using computational fluid dynamics instead of costly physical model tests (Christensen et.al. 2011).

Future work on the model and improvements to it include:

- Accounting for directional wave spreading. Directional spreading reduces the loads on the foundation (see Nielsen et.al. (2012) for more details).

- Make consistent corrections of the wave height distribution from linear simulations in order for them to emulate theoretical or empirical wave height distributions

- Investigate whether there are physical constraints on the load that needs to be considered

ACKNOWLEDGMENTS
The work has been partially funded by the ForskEL Project 2010-1-10495, Wave loads on offshore wind turbines.

REFERENCES


Figure 12. Hindcast model extension and position of data extraction point. Time series of $H_{m0}$ shown in insert.